PERIOD-II

M10CH4

TOPIC

4 Number Base

4.1 THE BASE TEN SYSTEM

Numbers are usually written in base 10 (decimal numeral) but they can be written or converted to other bases as well. Base 10 is called the decimal numeral.

To indicate that a base other than 10 is being used, a small subscript is added after the number. Hence 32_4 indicates that the number has been written in base 4.

Let us consider the number (or numeral) 574. We know that it is read as 'five hundred seventy four'. It can be written as

$$574 = 500 + 70 + 4$$
$$= 5 \times 100 + 7 \times 10 + 4 \times 1$$
$$= 5 \times 10^2 + 7 \times 10^1 + 4 \times 10^0$$

The place values (or weights) of digits *from right to left* are powers of 10, i.e., 10^0, 10^1, 10^2. The pupils are familiar with this system from earlier days of schooling. This system is called the *decimal system.* In this system, we use the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. We say that decimal system is a system with base ten. The base is written as a subscript. Thus, $574 = (574)_{10}$. Since the decimal system is the most commonly used, the base is usually not mentioned.

4.2 CONVERSION OF BASE 10 TO OTHER BASES

In fact, any integer greater than 1 can be taken as a base. In general, if the base is '*d*', we take the digits 0, 1, 2, 3, ..., *d* – 1 to represent the number. The system with smallest possible base 2 is called the *binary system.* In this system, the only two digits used are 0 and 1. The number $(101)_2$ in the binary system is read as 'one zero one' and not as 'one hundred one'. We read the digits one by one. The system with base 3 is called the *ternary system.* In this system, the only three digits used are 0, 1 and 2.

To represent a number in the binary system (base 2), the place values are taken to be powers of the base, *i.e.*, $2^0, 2^1, 2^2, 2^3, \ldots$ from right to left.

To represent a number in the ternary system (base 3), the place values are taken to be powers of the base, *i.e.,* $3^0, 3^1, 3^2, 3^3, \ldots$ from right to left.

Continuing like this, to represent a number in the system with base '*d*', the only '*d*' digits used are 0, 1, 2, 3, ..., $d - 1$ and the place values are taken to be powers of *d*, i.e., $d^0, d^1, d^2, d^3, \ldots$ from right to left.

Bases can be converted from other bases such as 2, 5, 8, 12 etc. to base 10 or vice versa.

In order to change from base 10 to a different base, a method involving *successive division* is used. The given decimal numeral is divided *repeatedly* by the appropriate base number, and the remainders, including zero, are noted at each stage. The division is continued until there is nothing left to divide. The answer is obtained by reading the remainders upwards as indicated by the arrows.

Example 1. *Convert 246_{10} to number base 8.*

Solution. Divide 246 repeatedly by 8.

8	246	
8	30	remainder 6
8	3	remainder 6
	0	remainder 3

∴ $246_{10} = 366_8$.

Example 2. *Convert 372_{10} to base eight.*

Solution. Divide 372 repeatedly by 8.

8	372	
8	46	remainder 4
8	5	remainder 6
	0	remainder 5

∴ $372_{10} = 564_8$.

Example 3. *Convert 1275_{10} to base twelve.*

Solution. Divide 1275 repeatedly by 12.

12	1275	
12	106	remainder 3
12	8	remainder 10
	0	remainder 8

∴ $1275_{12} = 8T3_{12}$.

Note: 10 is represented by T in base 12.

EXERCISE 4.1

1. Convert 7_{10} to number base 2 or binary numeral.
2. Convert 30_{10} to base four.
3. Convert 105 to number base two.

4.3 CONVERSION OF OTHER BASES TO BASE TEN

Converting from other bases to decimal numbers (i.e., base ten) is very simple, you remember that each digit in the other base number represents a power of that base number.

Example 4. *Convert 100101_2 to the corresponding base ten number.*

Solution. Write the digits in order, and count them from the Right to Left starting from zero:

1	0	0	1	0	1	: Digits
5	4	3	2	1	0	: Numbering

Use this listing to convert each digit to the power of two that it represents:

$1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 1 \times 32 + 0 \times 16 + 0 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1$

$= 32 + 0 + 0 + 4 + 0 + 1 = 37$

Thus, 100101_2 converts to 37_{10}.

Example 5. *Convert 342_8 to number base 10.*

Solution. Write the digits in order, and count them from Right to Left starting from zero:

3	4	2	: Digits
2	1	0	: Numbering

Use this listing to convert each digit to the power of eight that it represents:

$$342_8 = (3 \times 8^2) + (4 \times 8^1) + (2 \times 8^0)$$
$$= (3 \times 64) + (4 \times 8) + (2 \times 1)$$
$$= 192 + 32 + 2 = 226_{10} = 226$$

Thus, 342_8 converts to 226_{10}.

Example 6. *Find in base ten the value of the 2 in 123_5.*

Solution.

$$123_5 = (1 \times 5^2) + (2 \times 5^1) + (3 \times 5^0)$$
$$= (1 \times 25) + (2 \times 5) + (3 \times 1)$$
$$= 25 + 10 + 3$$

∴ The value of the 2 in base ten is 10.

Note: **1.** A decimal numeral is the same as base 10.

2. Any number raised to the power zero is 1. *i.e.* $5^0 = 1$, $8^0 = 1$.

EXERCISE 4.2

1. Convert 1011_2 to number base 10 (decimal numeral).
2. Convert 2321_4 to number base 10 (or decimal numeral).
3. Convert 232_5 to decimal numeral.
4. Convert (*a*) 11.001_2 (*b*) 110.11_2 to a decimal numeral (or base 10).
5. Convert (*a*) 123_5 (*b*) 110011_2 to a decimal numeral (or base 10).

4.4 OPERATION ON NUMBERS INVOLVING NUMBER BASES OTHER THAN BASE TEN

Addition *and* Subtraction

Addition and subtraction involving number bases other than base ten is same as the numbers of base ten. But you need to be very careful about 'carry' during addition and 'borrow' during subtraction. It must be according to the number bases taken for operation.

In addition of numbers other than base ten, we add the two numbers and convert the resultant number in base we are working in and then forward 'carry' to the next column. Look at these examples.

Example 7. *Evaluate:*

(*a*) $123_5 + 241_5$ (*b*) $3047_8 + 442_8$

Solution.

(*a*)

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ } \\ 1\ 2\ 3_5 \\ +\ 2\ 4\ 2_5 \\ \hline 4\ 2\ 0_5 \end{array}$$

Explanation. Start from right ($3 + 2 = 5 = 10_5$). So, we write 0 and carry 1 to the next column.

The next column becomes ($1 + 2 + 4 = 7 = 12_5$). So, we write 2 and carry 1 to the next column.

The next column becomes $1 + 1 + 2 = 4$.

(*b*)

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ } \\ 3\ 0\ 4\ 7_8 \\ +\ 4\ 4\ 2_8 \\ \hline 3\ 5\ 1\ 1_8 \end{array}$$

Explanation. Start from right ($7 + 2 = 9 = 11_8$). So, we write 1 and carry 1 to the next column.

The next column becomes ($1 + 4 + 4 = 9 = 11_8$). So, we write 1 and carry 1 to the next column.

The next column becomes $1 + 4 = 5$.

So, we write 5 and 3 after it.

In subtraction of numbers other than base ten, we borrow the number equal to the base we are working in. Look at these examples.

Example 8. *Evaluate:*

(*a*) $324_5 - 233_5$ (*b*) $713_8 - 414_8$

Solution.

(*a*)

$$\begin{array}{r} 3\ 2\ 4_5 \\ -\ 2\ 3\ 3_5 \\ \hline 4\ 1_5 \end{array}$$

Explanation. Start from right ($4 - 3 = 1$). For the second column, we borrow 5 and add it to 2 to make 7 ($5 + 2 = 7$).

Now subtract 3 from 7 ($7 - 3 = 4$), write 4 in second column.

In next column left with 2 subtract 2 ($2 - 2 = 0$), write nothing in this column.

(*b*)
$$\begin{array}{r} 7\ 1\ 3_8 \\ -\ 4\ 1\ 4_8 \\ \hline 2\ 7\ 7_8 \end{array}$$

Explanation. Start from right, borrow 8 from the second column and add it to 3 then subtract 4 (*i.e.* $8 + 3 - 4 = 7_8$). Write 7 below in this column.

In next column again borrow 8 from next column and add it to 0 and subtract 1 *i.e.,* $8 + 0 - 1 = 7$. Write 7 below in this column. In next column $6 - 4 = 2$. Write 2 below in this column.

EXERCISE 4.3

1. Evaluate: $234_5 - 141_5$.
2. Perform the following operations with explanation in number base 8.
   (*a*) 437 + 75 (*b*) 612 – 463.
3. Perform the following operations
   (*a*) $321_5 - 123_5$ (*b*) $411_5 - 202_5$.

4.5 MULTIPLICATION

The table for binary multiplication is

×	0	1
0	0	0
1	0	1

Thus, $0 \times 0 = 0$, $0 \times 1 = 0$
$1 \times 0 = 0$, $1 \times 1 = 1$.

Example 9. *Perform* $(1101)_2 \times (110)_2$ *and check your answer.*

Solution.

$$\begin{array}{r} 1\ 1\ 0\ 1_2 \\ 1\ 1\ 0_2 \\ \hline 0\ 0\ 0\ 0 \\ 1\ 1\ 0\ 1\ \times \\ 1\ 1\ 0\ 1\ \times\ \times \\ \hline 1\ 0\ 0\ 1\ 1\ 1\ 0_2 \\ \hline \end{array}$$

Thus, $(1101)_2 \times (110)_2 = (1001110)_2$

Verification:

$$(1101)_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$
$$= 8 + 4 + 0 + 1 = 13$$
$$(110)_2 = 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$
$$= 4 + 2 + 0 = 6$$
$$(1001110)_2 = 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$
$$= 64 + 0 + 0 + 8 + 4 + 2 + 0 = 78$$

and $13 \times 6 = 78.$

Hence the verification.

Example 10. *Perform* $354_8 \times 463_8$ *with explanation.*

Solution.

$$\begin{array}{r} 3\,5\,4_8 \\ 4\,6\,3_8 \\ \hline 1\,3\,0\,4 \\ 2\,6\,1\,0\,\times \\ 1\,6\,6\,0\,\times\,\times \\ \hline 2\,1\,5\,4\,0\,4_8 \\ \hline \end{array}$$

Explanation:

(*i*) $4 \times 3 = 12$ [12 divided by 8, remainder 4 and carry over 1]

$3 \times 5 + 1 = 15 + 1 = 16$

[16 divided by 8, remainder 0 and carry over 2]

$3 \times 3 + 2 = 9 + 2 = 11$

[11 divided by 8, remainder 3 and carry over 1]

Then, we get $354_8 \times 3_8 = 1304_8$.

(*ii*) $6 \times 4 = 24$ [24 divided by 8, remainder 0 and carry over 3]

$6 \times 5 + 3 = 30 + 3 = 33$

[33 divided by 8, remainder 1 and carry over 4]

$6 \times 3 + 4 = 18 + 4 = 22$

[22 divided by 8, remainder 6 and carry over 2]

Then, we get $354_8 \times 6_8 = 2610_8$.

(*iii*) $4 \times 4 = 16$ [16 divided by 8, remainder 0 and carry over 2]

$4 \times 5 + 2 = 20 + 2 = 22$

[22 divided by 8, remainder 6 and carry over 2]

$4 \times 3 + 2 = 12 + 2 = 14$

[14 divided by 8, remainder 6 and carry over 1]

Then, we get $354_8 \times 4_8 = 1660_8$.

Add:

$$\begin{array}{r} {}^{1}\,1\,3\,0\,4_8 \\ {}^{1}\,2\,6\,1\,0\,0_8 \\ 1\,6\,6\,0\,0\,0_8 \\ \hline 2\,1\,5\,4\,0\,4_8 \\ \hline \end{array}$$

EXERCISE 4.4

1. Perform the following:

(*a*) $102_3 \times 21_3$ (*b*) $2102_3 \times 122_3$ (*c*) $(11_3)^2$ (*d*) $(20_3)^2$.

4.6 OPERATION IN OTHER BASES

The methods for adding and subtracting in other number bases are exactly the same as for decimal numerals.

Example 11. *Perform the following:* $111001_2 - 10101_2$.

Solution.

$$\begin{array}{r} 1\,1\,1\,0\,0\,1_2 \\ -\,1\,0\,1\,0\,1_2 \\ \hline 1\,0\,0\,1\,0\,0_2 \\ \hline \end{array}$$

Example 12. *If the addition performed in base 4, find the missing number.*

$$\begin{array}{r} 3\,1\,0\,1 \\ +\,*\,*\,*\,* \\ 2\,1\,3 \\ \hline 1\,0\,3\,0\,1 \\ \hline \end{array}$$

Solution. For the missing number, add 3101 and 213 in base 4 and subtract the result from 10301 in base 4.

$$\begin{array}{r} 3\,1\,0\,1_4 \\ +\,2\,1\,3_4 \\ \hline 3\,3\,2\,0_4 \\ \hline \end{array} \quad \text{and} \quad \begin{array}{r} 1\,0\,3\,0\,1_4 \\ -\,3\,3\,2\,0_4 \\ \hline 3\,2\,1_4 \\ \hline \end{array}$$

Hence, the missing number is 321 in base 4.

EXERCISE 4.5

1. Perform the following: $11001_2 + 10111_2$.

2. Write the missing number if the following subtraction is in base 6.

$$\begin{array}{r} 4\,5\,2 \\ -\,*\,*\,* \\ \hline 2\,5\,4 \\ \hline \end{array}$$

3. Perform the following: $141_6 + 233_6 - 102_6$.

4.7 SIMPLE BASE EQUATIONS

To solve equations in which x occurs as the base of some numeral, reduce both sides to decimal system and then solve. Since x is a base, it must be a positive integer greater than 1.

Example 13. *Solve:* $(75)_x = (68)_{10}$.

Solution. Given $(75)_x = (68)_{10}$

Reducing the numeral on left side of the equation to base 10

$$(7 \times x^1 + 5 \times x^0)_{10} = (68)_{10}$$

$\Rightarrow \quad 7x + 5 = 68 \quad \Rightarrow \quad 7x = 68 - 5$

$\Rightarrow \quad 7x = 63 \quad \Rightarrow \quad x = 9$

Verification: $(75)_9 = 7 \times 9^1 + 5 \times 9^0$

$= 63 + 5 = 68.$

Example 14. *Solve:* $(132)_x = (42)_{10}$.

Solution. Given $(132)_x = (42)_{10}$

Reducing the numeral on left side of the equation to base 10

$$(1 \times x^2 + 3 \times x^1 + 2 \times x^0)_{10} = (42)_{10}$$

$\Rightarrow \quad x^2 + 3x + 2 = 42 \quad \Rightarrow \quad x^2 + 3x - 40 = 0$

$\Rightarrow \quad x^2 + 8x - 5x - 40 = 0 \quad \Rightarrow \quad x(x + 8) - 5(x + 8) = 0$

$\Rightarrow \quad (x + 8)(x - 5) = 0$

$\Rightarrow \quad x = -8, 5$

Since x is the base of a numerical, it cannot be negative.

Therefore, $x = 5$

Verification: $(132)_5 = 1 \times 5^2 + 3 \times 5^1 + 2 \times 5^0$

$= 25 + 15 + 2 = 42.$

Example 15. *Solve: $(631)_x = (409)_{10}$.*

Solution. Given $(631)_x = (409)_{10}$

Reducing the numeral on left side of the equation to base 10

$$(6 \times x^2 + 3 \times x^1 + 1 \times x^0)_{10} = (409)_{10}$$

$$\Rightarrow \quad 6x^2 \times 3x + 1 = 409$$

$$\Rightarrow \quad 6x^2 + 3x - 408 = 0$$

Dividing by 3

$$2x^2 + x - 136 = 0$$

$$\Rightarrow \quad 2x^2 + 17x - 16x - 136 = 0$$

$$\Rightarrow \quad x(2x + 17) - 8(2x + 17) = 0$$

$$\Rightarrow \quad (2x + 17)(x - 8) = 0$$

$$\Rightarrow \quad x = -\frac{17}{2}, 8$$

Since x is the base of a numeral, it cannot be negative.

Therefore, $x = 8$

Verification:

$$(631)_8 = 6 \times 8^2 + 3 \times 8^1 + 1 \times 8^0$$

$$= 384 + 24 + 1 = 409.$$

EXERCISE 4.6

Solve the following equations for x:

1. $(31)_x = (16)_{10}$
2. $(46)_x = (38)_{10}$
3. $(101)_x = (5)_{10}$
4. $(201)_x = (19)_{10}$
5. $(245)_x = (101)_{10}$
6. $(396)_x = (468)_{10}$

REVIEW EXERCISE

1. Convert 72_{10} to base five.
2. Convert 77 to number base two.
3. Convert 12.04_5.
4. Convert
   (*a*) 1204_5 (*b*) 11001_2
5. Mr. Henry had 3540_8 oranges. He sold 402_8 and 317_8 got spoilt. How many oranges were left? Leave your answer in number base ten.
6. Perform the following:
   (*a*) $21_3 \times 21_3$ (*b*) $1322_8 \times 13_8$

7. Evaluate: $311_4 + 213_4 - 332_4$ leaving your answer in base 10.

8. If 263 + 441 = 714, find the number base that has been used.

Solve the following equations for x:

9. $(105)_x = (54)_{10}$ **10.** $(123)_x = (38)_{10}$

MULTIPLE CHOICE QUESTIONS (MCQs)

1. Convert 104_{10} to a binary numeral.
(*a*) 1101000 (*b*) 1101100 (*c*) 1011010 (*d*) 1010100

2. Write 1101101_2 in base ten.
(*a*) 125 (*b*) 31 (*c*) 145 (*d*) 109

3. Convert 11001_2 to a decimal numeral.
(*a*) 7 (*b*) 6 (*c*) 14 (*d*) 25

4. Convert 206 to base five numeral.
(*a*) 3321_5 (*b*) 411_5 (*c*) 4011_5 (*d*) 1311_5

5. Express 87_{10} as a base five numeral.
(*a*) 3202_5 (*b*) 322_5 (*c*) 302_5 (*d*) 3022_5

6. Convert 134_5 to base ten numeral.
(*a*) 220 (*b*) 16 (*c*) 44 (*d*) 40

7. Express 57_{10} as a base two (binary) numeral.
(*a*) 100111_2 (*b*) 101011_2 (*c*) 11010_2 (*d*) 111001_2

8. Convert 320_5 to a base ten numeral.
(*a*) 25 (*b*) 77 (*c*) 85 (*d*) 86

RECAP AT A GLANCE

- Base 10 is called the decimal numeral.
- The numeration system in base two is referred to as the binary system.
- The system with base 3 is called the *ternary* system.
- 2 is the smallest possible base.
- Bases can be converted from other bases such as 2, 5, 8 etc. to base 10 or from base 10 to bases 2, 5, 8 etc.
- A decimal numeral is the same as base 10.
- Any number raised to the power zero is 1.
- To solve an equation, first change the base on both sides of the equation to base ten.

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